Differential Equations for Black Queen soil simulations

Equation 1, change in cellulose pool over time

Equation 2, change in glucose pool over time

Equation 3, change in amino acid pool over time

Equation 4, change in prototrophic species *p* biomass over time

Equation 5, change in auxotrophic species *a* biomass over time

Table 1: Model parameters, symbols and explanations.

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| Parameter | Symbol | Explanation |
| Time (generations) | *t* | Each time point is expressed as a species’ generation. For simplicity, this is the same for all species. |
| Cellulose (µM) | *C* | Cellulose acts as the carbon source for the system, with 2 mM added every 200 generations. |
| Glucose (µM) | *G* | Cellulose must first be catabolised to glucose by cellulase. Glucose is subsequently taken up by species for growth. |
| Cellulase maximum activity and half-saturation constant (per µM) | *Cmax* , *CKm* | Cellulase activity is based on Michaelis-Menten kinetics and performs as the soil bacterium *Bacillus brevis* (Singh and Kumar, 1998). Only cellulytic species produce cellulase. |
| Glucose units per Cellulose | *α* | Glucose yield from one molecule of Cellulose. |
| Amino Acids (nM) | *A* , *AKm* | Amino acids are produced by Prototrophs and secreted as a leaky function based on Monod kinetics. Auxotrophs are dependent on taking up amino acids for growth. |
| Transporter maximum activity and half-saturation constants (per µM) | *Tmax* , *TKm* | Transporters are necessary for uptake of glucose and, in the case of auxotrophs, amino acids. Tmax is consistent between species and is based on *E. coli* phosphotransferase (Ferenci, 1996). TKm differs between copiotrophs and oligotrophs. Oligotrophs have half the Km of copiotrophs, and thus double the affinity for growth substrates. |
| Maximum growth rate and growth yield efficiency on glucose (per µM) | *µmax* , *Yglu* | The maximum growth rate of a species. Oligotrophs have an order of magnitude lower *µmax* than copiotrophs, and thus grow slower. Yglu is based on optimal *E. coli* growth under continuous culture, where 57% of input glucose is converted to biomass (Kayser *et al*., 2005). |
| Maintenance energy | *m* | Maintenance energy is the burden associated with a species’ functional repertoire. The higher the *m*, the slower the overall growth rate. Cellulytic Prototrophs have the highest *m*, followed by Cellulytic Auxotrophs, Non-cellulytic Prototrophs. Non-cellulytic Auxotrophs have the lowest *m*. Oligotrophs also have a slightly higher *m* than copiotrophs due to their higher-affinity, ATP-dependent transporters. |
| Mortality (%) | *γ* | Mortality rate of each species per generation, arbitrarily set to 20%. |
| Species biomass (ng) | *Si* | Increase or decrease in a species’ biomass per generation is a function of the parameters that make up its functional repertoire, life strategy and the capacity for the community as a whole to produce public goods. |
| Cellulytic species biomass (ng) | *Sc* | Those species capable of producing cellulase. |
| Prototrophic species biomass (ng) | *Sp* | Those species capable of producing amino acids. |
| Auxotrophic species biomass (ng) | *Sa* | Those species incapable of producing amino acids. |